Nassau County Interscholastic Mathematics League

Contest #5 Answers must be integers from 0 to 999, inclusive. 2021 – 2022

No calculators are allowed.

Time: 10 minutes **Name: _____**

25 If $\sqrt{x}+\sqrt{x}=\sqrt{100}$, compute x.

26. Compute: $-10(\log_3 49)\left(\log_5 \sqrt{2}\right)(\log_7 81)\left(\log_8 \frac{1}{125}\right)$.

25. ☐ 26. ☐

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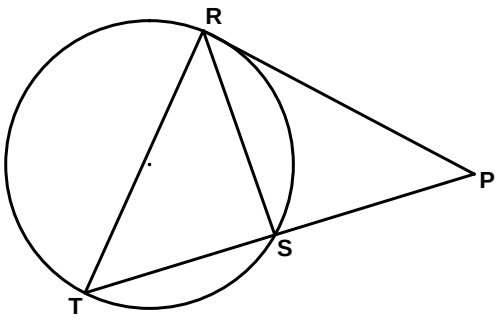
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Time: 10 minutes **Name: _____**

27. On a multiple choice test consisting of 600 questions, the correct answers follow the repeating pattern of BDACBDACBDAC… . A particular student's answer paper consists of the repeating pattern ABBCDABBCDABBCD … . Compute the number of answers that the particular student will get correct.

28. As in the given diagram, ΔRST is inscribed in a circle and $RS = 2$. The tangent line to the circle through point R meets $\overline{TS}$ extended through point S at point P, and point S is the midpoint of $\overline{PT}$. If, in simplest form, $RT = c\sqrt{k}$, compute ck.



27. ☐ 28. ☐

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Time: 10 minutes **Name: _____**

29. For all positive numbers x and y, $f(xy) = f(x) + f(y)$. If $f(2) = 13$ and $f(3) = 22$, compute $f(144)$.

30. Compute $300 \cdot \sum_{k=1}^{8} \frac{(k+1)\sqrt{k} - k\sqrt{k+1}}{k(k+1)}$.

29. ☐ 30. ☐

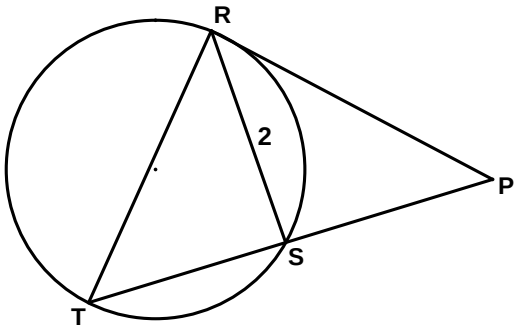
Solutions for Contest #5

25. $\sqrt{x}+\sqrt{x}=\sqrt{100}\rightarrow 2\sqrt{x}=10\rightarrow \sqrt{x}=5\rightarrow x=\mathbf{25}$.

26. Use the base change law and rearrange the bases to get
$-10(\log_7 49)\left(\log_8 \sqrt{2}\right)(\log_3 81)\left(\log_5 \frac{1}{125}\right)=$
$-10\left(\frac{2\log 7}{\log 7}\right)\left(\frac{0.5\log 2}{3\log 2}\right)\left(\frac{4\log 3}{\log 3}\right)\left(\frac{-3\log 5}{\log 5}\right)=-10\cdot 2\cdot\frac{1}{6}\cdot 4\cdot -3=\mathbf{40}$

27. For each set of 20 answers, the student has 5 correct answers. So, for 600 answers, the student will have **150** correct ones.

28. Since $m\measuredangle T=\frac{1}{2}m\widehat{RS}$ and $m\measuredangle PRS=\frac{1}{2}m\widehat{RS}$, then $\measuredangle T\cong\measuredangle PRS$. Also, since $\measuredangle P\cong\measuredangle P$, then $\Delta RSP\sim\Delta TRP$. Thus, $\frac{RS}{TR}=\frac{SP}{RP}=\frac{RP}{TP}$. Let $a=PS=ST\rightarrow\frac{2}{TR}=\frac{a}{RP}=\frac{RP}{2a}\rightarrow$ $2a^2=(RP)^2\rightarrow\frac{RP}{a}=\sqrt{2}=\frac{RT}{2}\rightarrow RT=2\sqrt{2}\rightarrow ck=\mathbf{4}$.



29. Note that $f(144)=f(16\cdot 9)=f(16)+f(9)=f(4\cdot 4)+f(3\cdot 3)=2f(4)+2f(3)=$ $2f(2\cdot 2)+2f(3)=4f(2)+2f(3)=4\cdot 13+2\cdot 22=\mathbf{96}$. Alternatively, note that f has a property of a log function, i.e. $\log AB=\log A+\log B$. Additionally, using $\log A^n=n\log A$, we have $f(144)=f(9\cdot 16)=f(9)+f(16)=$ $f(3^2)+f(2^4)=2f(3)+4f(2)=2(22)+4(13)=44+52=\mathbf{96}$.

30. $300\cdot\sum_{k=1}^{8}\frac{(k+1)\sqrt{k}-k\sqrt{k+1}}{k(k+1)}=300\cdot\sum_{k=1}^{8}\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}=300\cdot\left[\left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)+\cdots+\left(\frac{1}{\sqrt{8}}-\frac{1}{\sqrt{9}}\right)\right]=300\cdot\left(1-\frac{1}{3}\right)=\mathbf{200}$.